

# Factoring at the Individual Level: Some Matters for the Second Century of Factor Analysis

John R. Nesselroade  
University of Virginia

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# Individual Level Factor Analysis

Place of the individual in a SCIENCE of behavior is ambiguous

“...one traditional approach attempts to discover and delineate the general attributes of psychological phenomena as applicable to all individuals, while the alternate approach deals with individual differences in the extent or quality of selected aspects of behavior.” (Tucker, 1966)

# Individual Level Factor Analysis (con't.)

“individual differences in the **extent** or **quality** of ...behavior.”

Are we mere replicates (more or less) of each other or are there important unique features of each of us with which to reckon?

# Individual Level Factor Analysis (con't.)

I want to focus on three topics:

1. Aspects of the so-called idiographic-nomothetic debate as it bears on factor analyzing multivariate time-series of individuals.

# Individual Level Factor Analysis (con't.)

2. Reflections on the adequacy of the “received” view of factor invariance in the context of individual level factor analysis.

# Individual Level Factor Analysis (con't.)

3. Key role for extensions of individual level factor analysis for the future.

# Individual Level Factor Analysis (con't.)

Cattell's P—technique and *unitariness* of traits

“We should be very surprised if the growth pattern in a trait bore no relation to its absolute pattern, as an individual difference structure, and this would throw doubt on the scientific usefulness of the concept of unitariness for the trait.”



# Individual Level Factor Analysis (con't.)

But, Cattell expressed some misgivings:

“...whereas in the case of traits, the nature of the growth process may well be such that the growth weight pattern level  $x$  is significantly different from the individual differences pattern at level  $x$ , denoting that growth has previously occurred in a different way. For example, the height of the head (and presumably individual differences therein) contributes much more to the general stature factor in a young child than in an older adult, and this is an important fact in the biology of growth.”

# Cattell's P-technique Factor Analysis Model

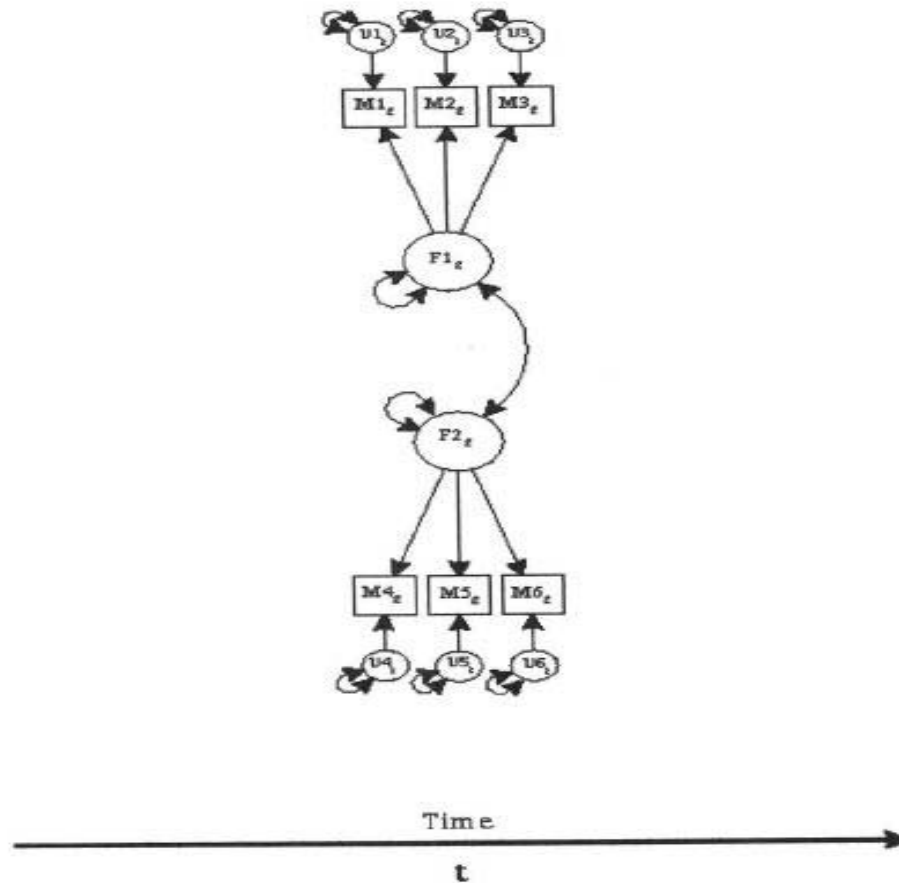
The P-technique model is specified as:

$$\mathbf{z}(t) = \Lambda(0)\boldsymbol{\eta}(t) + \boldsymbol{\varepsilon}(t)$$

where  $\mathbf{z}(t)$  is the observed or manifest  $p$ -variate time-series,  $\boldsymbol{\eta}(t)$  is the latent  $q$ -variate factor time-series,  $\boldsymbol{\varepsilon}(t)$  is a  $p$ -variate noise time-series, and  $\Lambda(0)$ , is a  $p \times q$  matrix of factor loadings.

# Individual Level Factor Analysis

## P-Technique Model



# Individual Level Factor Analysis (con't.)

P—technique was subject to criticisms—  
even by Cattell

-lack of lag structure in factors to  
variables relationships

# Individual Level Factor Analysis (con't.)

Molenaar (2004) has criticized the idea of convergent information from inter- and intra- sources using ergodic versus non-ergodic systems concepts. "...no asymptotic equivalence relationship between the structures of inter-individual differences and intra-individual variability exists" for non-ergodic processes.

# Individual Level Factor Analysis (con't.)

Bereiter (1963) weighed in with:

“A necessary and sufficient proof of the statement that two tests “measure the same thing” is that they yield scores which vary together over changing conditions. P-technique is the logical technique for studying the interdependencies of measures.”

**and warned that** “...correlations between measures over individuals should bear some correspondence to correlations between measures for the same or randomly equivalent individuals over varying occasions, and the study of individual differences may be justified as an expedient substitute for the more difficult P-technique.”

# Individual Level Factor Analysis (con't.)

Bereiter (1963) weighed in with:

Bereiter went on to say ``...however, the fact that measures on the same individuals may not correlate the same way on different occasions must be taken as evidence for the inadequacy of individual differences analysis as a substitute for P-technique analysis."

# Individual Level Factor Analysis (con't.)

For students of “process” such as developmentalists, the primary focus is on intra-individual variation and the key role of the differential approach is to identify individual differences and similarities in the patterns of intra-individual change rather than individual differences in putatively stable attributes.



# Place of the Individual in a Science of Behavior

Individual -- one of many replicates to provide variation, drive down the size of standard errors, etc., differing in amount but not in kind and providing a basis for inferring organization among variables versus

a unique entity, the repeated measurements of which provide the basis for inferring structures that define the organization of variables across occasions of measurement for that person; thus raising the question of how to aggregate information.

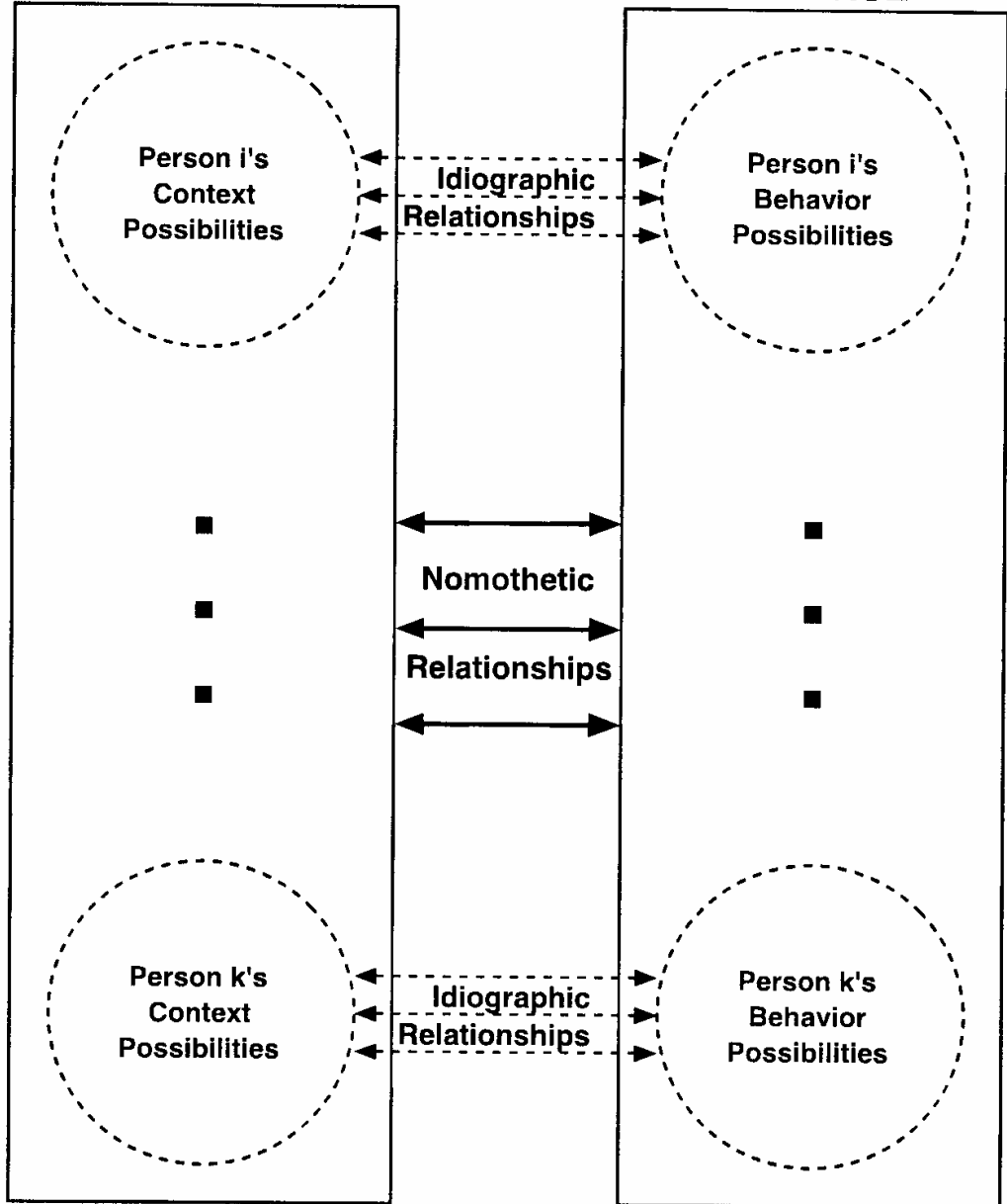
# Place of the Individual in a Science of Behavior (con't.)

## Idiographic versus Nomothetic relationships

“There are studies of individuals as such, an idiographic science. There are studies of people in groups or collectives, a nomothetic science. The analogue of a law of nature had no place in idiographic science while, perhaps with the help of statistical analysis, it could have a place in nomothetic science. The distinction is related to but not identical to that between two kinds of empirical inquiry. There is the intensive design, the study of a group by the study of a typical member and there is the extensive design, the aggregation of information about the members of a population into a statement about the population as a whole.” (Harre’ 2001)

**CONTEXT  
UNIVERSE**

**BEHAVIOR  
UNIVERSE**



# Place of the Individual in a Science of Behavior (con't.)

The point is that nomothetic relationships bear a major burden of “overriding” and synthesizing idiographic relationships which will tend to weaken or even suppress them.

# Place of the Individual in a Science of Behavior (con't.)

There have been several overt attempts to bring the idiographic and nomothetic approaches together

Lamiell's (1981) "idiothetic" approach

Zevon and Tellegen's (1982) use of P-technique

From a radically different framework, Mischel and colleagues now define "consistency" in personality in terms of patterns of intra-individual variability.

# Place of the Individual in a Science of Behavior (con't.)

Using individual level approaches such as P-technique and its variants to make more informed aggregations of information is a promising way to go.

The role of factor invariance is a linchpin in such efforts

# Factor Invariance

“Holy grail” of simple structure, confactor, and other rotation ideals and because of that the reason for countless graphical, topographical, and analytic rotation procedures.

In developmental research, for example, factor invariance has been used to make the critical distinction between qualitative and quantitative change.

# Factor Invariance (con't.)

Major focus is on loading patterns but not necessarily exclusively.

Loadings are our worldly windows into latent space--often used to decide if we are “measuring the same thing” in different instances--if loadings are invariant the nature of the variables is taken to remain intact across comparisons



# Factor Invariance (con't.)

$$z_{ji} = a_{j1}f_{1i} + a_{j2}f_{2i} + \dots + a_{jk}f_{ki} + u_{ji}$$

$$z'_{ji} = a_{j1}f'_{1i} + a_{j2}f'_{2i} + \dots + a_{jk}f'_{ki} + u'_{ji}$$

where  $z_{ji}$  represents an observed score for person  $i$  on variable  $j$  at some point in time,  $z'_{ji}$  represents an observed score for person  $i$  on variable  $j$  at some subsequent point in time.  $a$ 's are factor loadings,  $f$ 's are common factor scores, and  $u$ 's are unique factor scores.

# Factor Invariance (con't.)

Loading invariance allows the subtraction and rearrangement of terms into:

$$z'_{ji} - z_{ji} = a_{j1}(f'_{1i} - f_{1i}) + a_{j2}(f'_{2i} - f_{2i}) + \dots \\ + a_{jk}(f'_{ki} - f_{ki}) + (u'_{ji} - u_{ji})$$

Thus rendering the observed changes (or differences) parsimonious functions of changes (or differences) at the factor level.

# Factor Invariance (con't.)

But, when we enter the world of individual factor analysis, for the purpose of informed aggregation of information across individuals, I am uneasy about this line of argument regarding invariance.

Factor analysis needs to become a means to an end, not an end in itself.

# Factor Invariance (con't.)

At what level should aggregation occur?

- Lagged covariance matrices

- Factor solutions (invariance key)

# Factor Invariance (con't.)

*Must invariance necessarily be defined at the primary factor level?*

We have learned from experience, especially in the context of self-report that the nature of manifest variables can differ in surprising ways across individuals--

e.g., Mittenness & Nesselroade (1987)

## Factor Invariance (con't.)

*Must invariance necessarily be defined at the primary factor level?*

The earlier quote by Cattell that “the height of the head ...contributes more to the general stature factor in a young child than in an older child” implies that stature is not the “same” observed variable in critical ways.

# Factor Invariance (con't.)

*Must invariance necessarily be defined at the primary factor level?*

Indicators of autonomic nervous system activity such as respiration rate, blood pressure, perspiration rate, etc., are also subject to individual learning and conditioning histories.

# Factor Invariance (con't.)

*Must invariance necessarily be defined at the primary factor level?*

Therefore, to what extent can one expect the results of individual level factor analyses to be commensurate from one person to another?



# Factor Invariance (con't.)

*Must invariance necessarily be defined at the primary factor level?*

How does one proceed with building a nomothetic science when the information contributed from different individuals cannot be regarded as on the same footing?

# Primary Factor Loadings as Idiographic “Filters”

Even if manifest variables are not “the same” we can conceptualize primary factors as “the same,” given acceptable evidence.

Consider disease states such as cancer. The person with cancer A manifests a different (perhaps overlapping) set of symptoms than the person with cancer B but this does not deter our referring to both individuals as cancer victims.

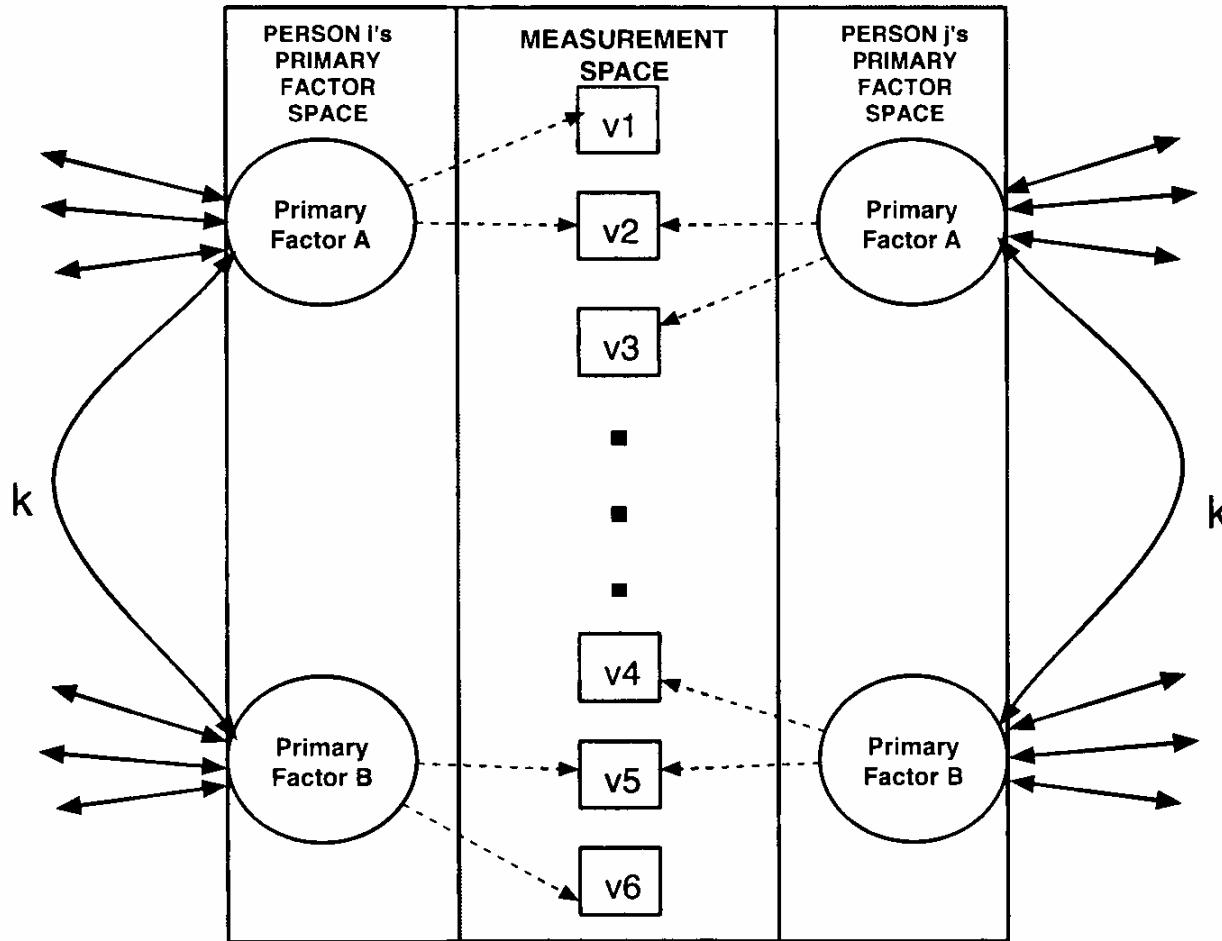
# Primary Factor Loadings as Idiographic “Filters”

At a psychological level, state anxiety connotes intra-individual variability but its relationships to indicators (self-report, autonomic variables, etc.) may well differ across persons due to “accidents” of heredity, personal history, and so on so factor invariance will not be observed.

# Primary Factor Loadings as Idiographic “Filters”

I want to construe factor loading patterns as filters that allow for idiosyncrasies at the individual-measures interface.

**WARNING!! WHAT YOU ARE ABOUT TO SEE IS NOT A PATH DIAGRAM!  
RATHER, IT IS A DIAGRAM WITH SOME PATHS!**



# Primary Factor Loadings as Idiographic “Filters”

An illustration (not evidence) from Lebo &  
Nesselroade (1978)

Table 1: Correlations Among Factors For Five Individuals: Cell entries are individual outcomes when the two factors were identified for a given subject. Inconsistencies are italicized.

	Energy	Well-Being	Fatigue	Social Affection	Well-Being(2)
Energy	--				
Well-Being	<i>+.35</i> , <i>+.60</i> , <i>+.76</i> , <i>-.35</i> , <i>+.66</i>	--			
Fatigue	<i>-.50</i> , <i>-.58</i> , <i>—</i> , <i>-.58</i> , <i>-.56</i>	<i>-.18</i> , <i>-.31</i> , <i>—</i> , <i>+.37</i> <i>-.42</i>	--		
Social-Affection	<i>+.48</i> , <i>-.04</i> , <i>+.32</i> , <i>—</i> , <i>—</i>	<i>+.65</i> , <i>+.25</i> , <i>+.46</i> , <i>—</i> , <i>—</i>	<i>-.08</i> , <i>+.15</i> , <i>—</i> , <i>—</i> , <i>—</i>	--	
Well-Being(2)	<i>+.31</i> , <i>+.68</i> , <i>+.30</i> , <i>+.70</i> , <i>—</i>	<i>+.52</i> , <i>+.61</i> , <i>+.53</i> , <i>-.02</i> <i>—</i>	<i>-.30</i> , <i>-.39</i> <i>—</i> , <i>-.50</i> , <i>—</i>	<i>+.49</i> , <i>+.14</i> , <i>+.32</i> , <i>—</i> <i>—</i>	--

# Primary Factor Loadings as Idiographic “Filters”

Despite all the “individuality” of the loading patterns, there is enough consistency in the factor inter-relations to suggest they might support an invariance interpretation at the second order (or higher) level.

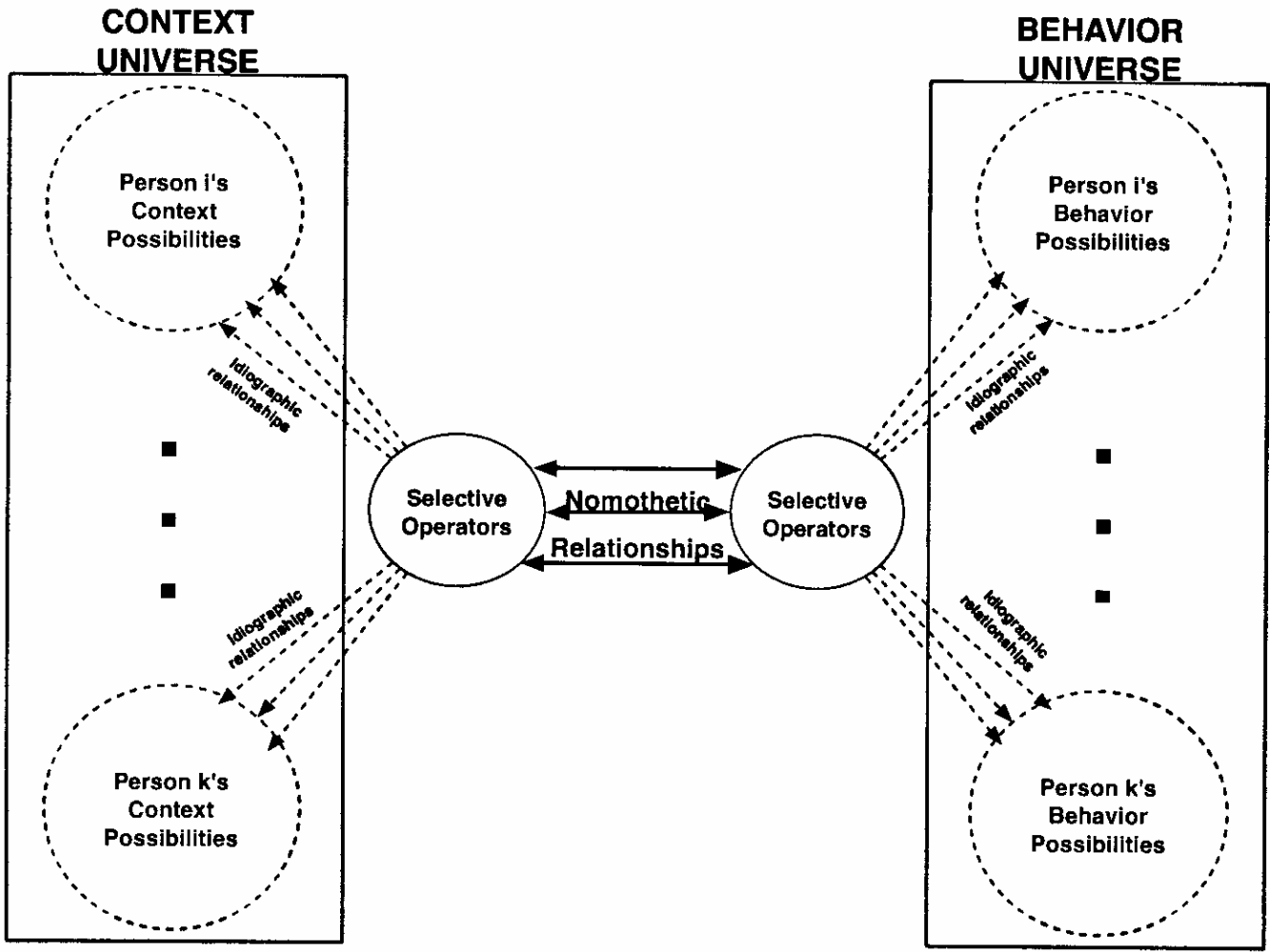


# Primary Factor Loadings as Idiographic “Filters”

In their presentation of *construct validity* Cronbach and Meehl (1955) argued that a necessary condition for a construct to be scientifically admissible was that it occur in a nomological net, at least some of whose laws involve observables.

# Primary Factor Loadings as Idiographic “Filters”

Elsewhere (Nesselroade & Ritter, 2001) I am trying to cast this situation in terms of selective operators and their possible mathematical group structure.



But, in the meantime,  
individual level factor analyses  
of multivariate time series  
information seems an  
immediately available path to  
identifying such patterns.

# Concluding Remarks

Factor analysis and some of its most cherished concepts such as factorial invariance have been instrumental in propelling the study of behavior to its present day level. We have enjoyed hearing about it for two days with more to come!

Included among those important innovations is the adaptation of the factor model to individual level analyses (e.g., trait-state distinction, framing of idiographic-nomothetic debate). Michael Browne will be focusing on some of these innovations.

# Concluding Remarks (con't.)

But, all features of the model of the past 100 years should not be slavishly applied to the study of intra-individual variability and change.

Aggregation and generalizability are key matters of concern for individual level modeling, which is where the idiographic-nomothetic debate is currently centered. But, working toward an “average” individual level representation does not seem to be the answer.

# Concluding Remarks (con't.)

In my view, the most promising attempts so far at resolving the idiographic-nomothetic debate involve capitalizing on the strengths of both individual level analyses and more traditional inter-individual differences approaches once the matter of who and what to aggregate is settled.

# Concluding Remarks (con't.)

Acknowledging the idiosyncrasies of individuals without letting those idiosyncrasies wreck the search for general lawfulness will require some innovation in our applications of basic notions of analysis. Until something better comes along, it is my belief that factor analysis at the individual level along with expanded versions of key concepts such as factor invariance can be a powerful and productive way to begin the method's second century.